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Theoretical study and software design for the implementation of a numerical scheme based on the Dual Time Stepping in COSMO LM

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SUMMARY This research paper describes a theoretical study for the re-design of the time integration core of COSMO based on a time integration technique called Dual Time Stepping. This technique has been successfully adopted in Aerodynamics for the resolution of the unsteady Navier-Stokes equations, but no previous attempt has been found in Metereology and Climatology. This preliminary theoretical study aims to assess the feasibility of implementing the Dual Time Stepping in the time integration core of COSMO. After the description of the Dual Time Stepping, the procedure to be implemented is described in comparison with the current time integration core. The typical convergence acceleration techniques (i.e. local time stepping, preconditioning, residual smoothing) that can be combined with the Dual Time Stepping are presented. A simple test case with a mountain flow is used to obtain a preliminary comparison between the re-designed and the existing software. This theoretical study will be followed by an assessment of the proposed numerical scheme that will space from stability considerations and implementation issues to the analysis of ideal and real test case for Meteorology and Climatology.

The work here presented is carried out under the technical supervision of ing. Pier Luigi Vitagliano (CIRA).

INTRODUCTION AND MOTIVATIONS

During years, one of the main goal in aerodynamics has been the research of efficient strategies for the time integration of the unsteady Navier-Stokes equations. Among several possibilities, Jameson's Dual Time Stepping (DTS) method [2] is widely used. This method performs time integration by achieving successive steady states in a fictious dual time. As pointed out in [4], a drawback of DTS is that steady-state iterations must be fully converged to guarantee a certain time accuracy, thus requiring a large number of dual-time iterations. The reason for the success of DTS is that it allows to exploit in unsteady calculations all the convergence acceleration techniques that are typically used for steady calculations, such as i) multigrid [5] ii) residual averaging [5, 6] and iii) local time stepping [7]. The stability and robustness of the dual-time stepping scheme has also been verified by other researchers.

No evidence of a previous attempt to use DTS in Numerical Weather Prediction (WPN) has been found in literature by the author. The aim of the present work is to assess the use of the DTS technique in Meteorology and Climatology, by its implemention in the COSMO code [1].

Additionally i) the increase of the horizontal resolution in the computational grids, ii) the presence of the orography and the convective effects that lead to a less pronunced distinction among the horizontal and vertical modes and iii) the use of the meteorological softwares for climatic forecasts characterized by extremely long integration times are paving the way to the use of implicit time-integration schemes. Hence the use of a DTS time integration scheme, characterized by reduced limitations on the size of the physical time step [2], seems to be a well-suited candidate in challenging the previous limitations and motivates the present research.

This research paper begins with a brief description of the mode-splitting [10] time advancement currently used in COSMO, which adopts different time advancement for the fast and slow processes in the governing equations. Subsequently the DTS technique is presented in details in conjunction with the acceleration techniques that will be implemented in the novel time integration core of COSMO. The software redesign for the implementation of a numerical scheme based on the DTS will be shown in comparison with the current algorithm flowchart, in order to evidence the substancial differences among the procedures. Before the conclusions, a comparative example among the current time integration scheme and the DTS scheme will be examined in details.

THE MODE-SPLITTING

Nonhydrostatic compressible models are of increasing interest because of their suitability for atmospheric simulations over a very wide range of meteorological phenomena from planetary down to local scales. Although simplistic at first glance, considerable difficulties are concealed in this equation set concerning forward-in-time integration methods. All possible atmospheric wave types are involved. Among these there are acoustic waves and gravity waves as fast wave types, on one hand, and slow advective processes connected on the other hand. The existence of acoustic waves is an obstacle to applying an overall explicit forward time schemes because it requires very small time increment for a stable integration process. From the meteorological point of view this wave type is unimportant and energetically irrelevant, and from the point of view of an efficient and economical numerical scheme, it is a nuisance to be damped out. Furthermore it is important that gravity wave processes and their physical relevance are retained as exactly as pos-

sible. A common strategy for an efficient numerical scheme in models involving fast and slow processes is the application of an explicit time-splitting method as is known since Marchuk's work [8], who first introduced it in the NWP in the field of nuclear physics. On this ground then, another pioneering work was the particularly efficient application of the splitting approach to the nonhydrostatic compressible equations by Klemp and Wilhelmson [9].

The basic idea of the splitting scheme, namely using a sufficiently small time step only for the fast subprocesses and a larger one for the slow model processes, was then further refined and investigated for the nonhydrostatic compressible model system in several papers and overview articles recently [10, 11]. The authors demonstrate stability analyses for their splitting schemes with increasing attention for forward time schemes.

Meanwhile, different nonhydrostatic models exist as research and operational weather forecasting models in which the splitting method with different modifications has been applied. Among these models there are the fifth-generation Pennsylvania State University-National Center for Atmospheric Research (NCAR) Mesoscale Model [13], the Advanced Research Weather Research and Forecast (WRF)-NCAR model [14], the Advanced Regional Prediction System [16], and COSMO of the German Weather Service [17] also used in Climatology.

In COSMO the prognostic equations for the dynamical variables, spherical wind components (u, v and w), and deviations of temperature and pressure T', p' from a base state, are splitted into a slow and a fast part. The slow part consists of the advection and Coriolis terms and tendencies from the physical processes. The fast parts are the pressure gradient terms and the working terms in the T'- and p'-equation, thus leading to sound expansion, and the buoyancy terms, leading to the expansion of gravity waves. In the splitting idea of Wicker and Skamarock [18] the tendency of the slow processes is calculated and added in each sub step of the fast processes.

TIME INTEGRATION

To illustrate the mode-splitting time integration scheme used in COSMO, we consider the model equations in the symbolic form:

$$\frac{\partial \underline{\mathbf{U}}}{\partial t} = s_{\underline{\mathbf{U}}} + f_{\underline{\mathbf{U}}} \qquad (1)$$

where $\underline{\mathbf{U}}$ denotes the prognostic model variables, $f_{\underline{\mathbf{U}}}$ the forcing terms due to the slow modes and $s_{\underline{\mathbf{U}}}$ the source terms which are related to the acoustic and gravity wave mode (i.e. fast modes).

Most time-split nonhydrostatic NWP models (e.g. ARPS, COAMPS, MM5, COSMO) use a leapfrog time discretization for the slow modes, and dissipation terms are commonly integrated forward in time (see [10]). Many semi-implicit models also use leapfrog time integration for the explicit (slow-mode) terms, although a number of these have switched to forward-in-time integration schemes. COSMO initially was used with a leapfrog time discretization but in recent years the Runge-Kutta schemes have been widely adopted. Its RK3 scheme, developed in [18] as the basis for the time-split scheme, is not a standard Runge-Kutta scheme per se because, while it is third-order accurate for linear equations, it is only second-order accurate for nonlinear equations [19]. The scheme is, however, more easily adaptable for stable time-splitting than other Runge-Kutta variants. The RK3 scheme circumvents three problems inherent in the leapfrog scheme. First, while the leapfrog scheme is second-order accurate, the scheme requires temporal filtering to prevent decoupling of the timesteps which reduces

the accuracy to first order. The RK3 scheme does not require this filtering. Second, the leapfrog scheme possesses large phase errors compared to the RK3 scheme. Third, the RK3 scheme allows both neutral and dissipative (upwind-biased) spatial discretizations for advection, whereas leapfrog is stable only for centered (neutral) operators.

Defining the prognostic variables in the COSMO solver as $\underline{\mathbf{U}} = (u, v, w, p', T')$ and the model equations as

$$\frac{\partial \underline{\mathbf{U}}}{\partial t} = -R(\underline{\mathbf{U}}) \tag{2}$$

where $R(\underline{\mathbf{U}})$ denotes the rigth hand side of the model equation for $\underline{\mathbf{U}}$, the RK3 integration takes the form of three steps to advance a solution $\underline{\mathbf{U}}(t)$ to $\underline{\mathbf{U}}(t + \Delta t)$:

$$\begin{aligned} \underline{\mathbf{U}}^{*} &= \underline{\mathbf{U}}^{t} - \frac{\Delta t}{3} R(\underline{\mathbf{U}}^{t}) \\ \underline{\mathbf{U}}^{**} &= \underline{\mathbf{U}}^{t} - \frac{\Delta t}{2} R(\underline{\mathbf{U}}^{*}), \\ \underline{\mathbf{U}}^{t+\Delta t} &= \underline{\mathbf{U}}^{t} - \Delta t R(\underline{\mathbf{U}}^{**}) \end{aligned} \tag{3}$$

where Δt is the time step for the slow-frequency modes (the model time step), and superscripts denote time levels.

As stated before, the high-frequency acoustic modes are meteorologically insignificant [15] and would severely limit an explicit RK3 time step Δt. To circumvent this time step limitation COSMO uses a time-split version of the RK3 scheme [18]. The time-splitting technique involves integrating terms associated with acoustic modes with smaller time-steps than the low-frequency (meteorologically significant) modes. Within the small timestep integration, terms associated with horizontally propagating modes are integrated explicitly, while terms associated with vertically propagating modes are integrated implicitly. The implicit integration component alleviates the severe Courant number restriction, arising from vertically propagating acoustic modes when using grids with large aspect ratios $\Delta x/\Delta z$, at the cost of a simple tridiagonal matrix inversion.

The Figure 1 shows the three stages of the Runge-Kutta time integration procedure, where the blue arrows represent the small time steps used to advance the fast tendencies of the governing equations, while the slow tendencies are kept constant in each RK3 stage, represented by the red arrows.

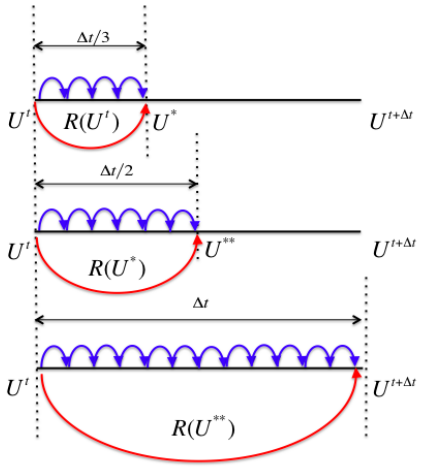

Figure 1:
The fast processeses are represented by the blue advancements using a small time step, while the slow processes are represented by the red advancements using the time step given by the RK3 scheme

Source: RK3 Time Integration

DUAL TIME STEPPING (DTS)

The DTS method considers the flow solution at each time step by recasting the governing equations as a pseudo steady problem, which is solved by integrating over an unphysical parameter called dual time. One of the main advantages deriving by the use of the DTS is that it can be easily adapted to pre-existing steady state solvers and all the convergence acceleration techniques (multigrid, residual averaging,

local time stepping, etc.) can be reused without significant software modifications. This has been the key success factor of the DTS method in Aerodynamics where the steady solvers preceded the unsteady ones, while in Meteorology the unsteady problems were directly challenged.

The semidiscrete equation for the prognostic variables ($\underline{\mathbf{U}}$) can be written in the implicit form:

$$L_t \underline{\mathbf{U}}^{t+\Delta t} = -R(\underline{\mathbf{U}}^{t+\Delta t}), \tag{4}$$

where L_t represents a time derivative operator. A second-order backward difference formula is used for time discretization

$$L_t \underline{\mathbf{U}}^{t+\Delta t} = \frac{3\underline{\mathbf{U}}^{t+\Delta t} - 4\underline{\mathbf{U}}^{t} + \underline{\mathbf{U}}^{t-\Delta t}}{2\triangle t}. \tag{5}$$

By indicating with k the dual iteration index, the DTS considers the residual as follows:

$$\underline{\mathbf{R}}^{\star k} = R(\underline{\mathbf{U}}^{k}) + \frac{3\underline{\mathbf{U}}^{k} - 4\underline{\mathbf{U}}^{t} + \underline{\mathbf{U}}^{t-\Delta t}}{2\triangle t}. \tag{6}$$

We assume in the following that $R(\underline{\mathbf{U}}^{k})$ is evaluated using the same equations implemented in the current time integration core of COSMO. The residual $\underline{\mathbf{R}}^{\star k}$ is such that

$$\lim_{k\to\infty} \underline{\mathbf{R}}^{\star k} = 0. \tag{7}$$

As a consequence, the solution at each time step of the equation

$$\underline{\mathbf{R}}^{\star k} = 0 \tag{8}$$

provides the unknown $\underline{\mathbf{U}}^{t+\Delta t}$. A pseudo-time derivative is introduced in such a way that

$$\frac{\partial \underline{\mathbf{U}}}{\partial \tau} = -\underline{\mathbf{R}}^{\star k} \tag{9}$$

The pseudo time τ is a relaxation parameter and has no physical meaning, not affecting the converged solution (see Figure 2).

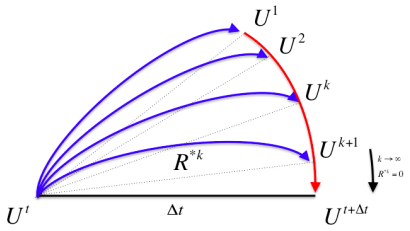

Figure 2:
The blue advancements represents the steps in a fictious time and are performed with a Runge-Kutta scheme. The modified residual (R^{*k}, including the dual time derivative) is updated each dual time step with the extimated value of the prognostic variables (U^k). The red arrow represents the convergence of the variables to a fictitious dual steady state corresponding to the physical time level $t + \Delta t$, identified by the condition $R^{*k} = 0$.

Source: DTS Time Integration

DTS WITH EXPLICIT INNER RELAXATION

The advantage of using the dual time is that the convergence toward the steady state does not require time accuracy, so any technique for accelarating the convergence can be adopted. Dual time integration is performed by using a Runge-Kutta scheme. Denoting with the subscripts i, j, k a particular grid point of the integration domain, an explicit stage of the Runge Kutta scheme with a local time step $\triangle\tau_{ijk}$ applied to equation (9) reads

$$\underline{\mathbf{U}}^{k+\frac{m}{q}} = \underline{\mathbf{U}}^{k} - \alpha_m \triangle\tau_{ijk}\, \underline{\mathbf{R}}^{\star\left(k+\frac{m-1}{q}\right)} \quad m = 1...q, \tag{10}$$

where q is the number of stages and α_m is the coefficient of the $m-$stage. By expliciting the definition of the residual $\underline{\mathbf{R}}^{\star}$ in equation 6 we have

$$\underline{\mathbf{U}}^{k+\frac{m}{q}} = \underline{\mathbf{U}}^{k} + \\ -\alpha_m \triangle\tau_{ijk}\left(\frac{3\underline{\mathbf{U}}^{k+\frac{m-1}{q}} - 4\underline{\mathbf{U}}^{t} + \underline{\mathbf{U}}^{t-\Delta t}}{2\triangle t} + \right. \\ \left. + R(\underline{\mathbf{U}}^{k+\frac{m-1}{q}})\right) \tag{11}$$

According to Melson [20], if the term $\underline{\mathbf{U}}^{k+\frac{m-1}{q}}$ appearing on the right-hand side of equation 11 is evaluated at the unknown stage m, the stability restrictions of the scheme are alleviated. Thus, we have

$$\left(1+\frac{3\triangle\tau_{ijk}}{2\triangle t}\,\alpha_m\right)\underline{\mathbf{U}}^{k+\frac{m}{q}}=\underline{\mathbf{U}}^{k}+ \\ -\alpha_m\triangle\tau_{ijk}\left(\frac{-4\underline{\mathbf{U}}^{t}+\underline{\mathbf{U}}^{t-\Delta t}}{2\triangle t}+R(\underline{\mathbf{U}}^{k+\frac{m-1}{q}})\right). \tag{12}$$

Finally, by adding and subtracting the same term evaluated at stage $m-1$ the expression of $\underline{\mathbf{R}}^{\star}$ on the right-hand side is recovered. Equation 12 becomes:

$$\left(1+\frac{3\triangle\tau_{ijk}}{2\triangle t}\alpha_m\right)\underline{\mathbf{U}}^{k+\frac{m}{q}}=\underline{\mathbf{U}}^{k}+ \\ +\frac{3\triangle\tau_{ijk}}{2\triangle t}\alpha_m\underline{\mathbf{U}}^{k+\frac{m-1}{q}}-\triangle\tau_{ijk}\alpha_m\underline{\mathbf{R}}^{\star\left(k+\frac{m-1}{q}\right)}. \tag{13}$$

LOCAL TIME STEPPING

The use of local time steps [7] allows the signals propagate at speeds in proportion to cell sizes. For steady state calculations, a faster expulsion of disturbances can be achieved by locally using the maximum allowable time step. Using this accelaration technique, the local time step $\Delta\tau_{ijk}$ is computed based on the local dimension of the grid, the magnitude of the local velocity and the Courant-Friedrichs-Lewy (CFL) number.

IMPLICIT RESIDUAL AVERAGING

This technique was first introduced by Lerat [24] and later implemented on the Runge-Kutta Stepping scheme by Jameson [25] in order to extend the stability limit of the scheme. CFL number is increased by replacing the residual at each cell by a weighted average of residuals at neighboring cells. This is done at each stage of the Runge-Kutta Stepping scheme before the solution is updated. Linear stability analysis has shown that the Runge-Kutta scheme with implicit residual smoothing may be made unconditionaly stable provided that the parameter used for the smooting, $RESAV$, is sufficiently large [25]. As long as $RESAV$ is not too large the use of implicit residual smoothing does not upset the formal second-order accuracy in space. Since the smoothing is done in space it has no effect on the temporal accuracy of the scheme. To date implicit residual smoothing has been used in conjunction with a spatially varying time step to accelerate convergence to a steady state.

PRECONDITIONING

Preconditioning is simply a mathematical tool used to remove stiffness from ill-conditioned systems of equations and involve the alteration of the time-derivatives used in time-marching CFD methods with the primary objective of enhancing their convergence. The main difficulty associated with the numerical simulation of the unsteady equations is the degradation of convergence at very low flow speeds: there is a huge disparity between the convective and acoustic eigenvalues in this system of equations because the speed of sound is much larger than the convective velocity. Hence, time step size in time marching schemes is limited by acoustics instead of convection, even though the former is not relevant in most low speed flows (Turkel [21]). Premultiplying the time derivative changes the eigenvalues of the system and accelerates the convergence to steady state:

$$\underline{\underline{\Gamma}}\cdot\frac{\delta\underline{\mathbf{U}}}{\delta\tau}=R(\underline{\mathbf{U}}) \tag{14}$$

where $\underline{\underline{\Gamma}}$ is a preconditioning matrix as defined in Turkel [22, 23].

MULTIGRID

Multigrid techniques [3, 2] allow global error to be addressed by using a sequence of successively coarser meshes. This method is based upon the principle that global (low-frequency) error existing on a fine mesh can be represented on a coarse mesh where it again becomes accessible as local (high-frequency) error: the clear advantage is that the numerical schemes are more efficient in damping high frequency errors [27, 28, 29].

BOUNDARY CONDITIONS

In this section a short description of the boundary conditions adopted by COSMO is presented. Non-penetrative boundary conditions are imposed at the *upper boundary* that is considered a rigid lid by setting the contravariant vertical velocity to zero. Since the *lower boundary* (that follows the surface terrain) is non-penetrative with respect to grid-scale mass fluxes, the contravariant vertical velocity must vanish. Here for the components of the horizontal velocity, the temperature and the water substances, friction boundary conditions are imposed.

In COSMO enhanced damping near the top of the model domain (i.e. damping layer) is accomplished by Rayleigh friction terms which are added to the right hand side of the prognostic equations for momentum, temperature and pressure perturbation. Thus, the COSMO formulation of the damping layer tends to restore the externally specified boundary fields near the top of the domain.

When we use a regional model for NWP purposes, information on the variables at the lateral boundaries and their time evolution must be specified by an external data set. These external data may be obtained by interpolation from a forecast run of another model or from a coarser resolution run of COSMO. Time dependent relaxation boundary conditions can then be used to force the solution at the lateral boundaries using the external data. Nesting a high-resolution limited area model in a low-resolution driving model causes numerical problems, since the time evolution of the model variables is based on a system of equations that can differ from that of the driving model. This leads to the generation of numerical noise, which can propagate from the lateral boundaries inward to the centre of the model domain. A simple and effective solution to this problem is to apply a sponge to the model variables within a *relaxation zone* close to the boundaries. In this zone, the variables of the high- resolution model are gradually modified to blend them with the driving model variables. In this way, the information transfer problem is cured, since information near the lateral boundaries is no longer generated by the high-resolution model but determined by the values of the low-resolution driving model.

The *periodic boundary* condition assumes that the solution of the model equations replicates itself indefinitely outside of the computational domain. Thus, the solution at a distance d to the west (north) of the computational domain western (northern) boundary equals the solution at the same distance d to the west (north) of the eastern (southern) boundary. Further informations on the treatment of the boundary conditions can be found in the implementation details of the software [1].

IMPLEMENTATION DETAILS OF THE RUNGE-KUTTA SCHEME

The following steps are performed in the routine **src_runge kutta.f90** of COSMO:

1. At the beginning the tendencies of the

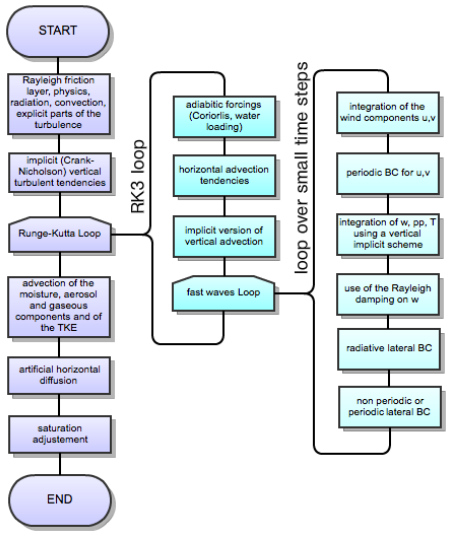

Figure 3:
Original design of the COSMO time integration core

Source: RK3 Flow Chart

Rayleigh friction layer [1] and the tendencies of the physical parameterizations of radiation, convection and the explicit parts of the turbulence (if activated) are summed up

2. The implicit (i.e. using a Crank-Nicholson scheme) vertical turbulent tendencies are added to the previously evaluated tendencies

3. Start of the Runge-Kutta substeps:

   (a) calculate the horizontal advection tendencies

   (b) if the implicit vertical advection is used then, using the previously calculated tendencies from horizontal advection and the above mentioned tendencies due to physics and adiabatic processes of the dynamic variables, the vertical advection is solved by a vertically implicit scheme.

   (c) The complete slow tendencies are inserted into the fast waves solver

4. end of the Runge-Kutta substeps.

5. Advection of the moisture fields (q_x), aerosol, gaseous components and the Turbulent Kinetic Energy (TKE) if active

6. In an operator splitting manner perform the horizontal diffusion of q_x and the implicit vertical diffusion of q_v (specific water vapor content), q_c (specific cloud water content), q_i (specific cloud ice content) and TKE

7. apply artificial horizontal diffusion [1] to the prognostic variables

8. call the saturation adjustment [1]

DTS REDESIGN OF THE TIME INTEGRATION CORE

In this section a redesign of the time integration core of COSMO is proposed with comparison to the previosly shown scheme. The following steps are performed in the novel routine **src_DTS_integration.f90** of COSMO:

1. At the beginning the tendencies of the Rayleigh friction layer and the tendencies of the physical parameterizations of radiation, convection and the explicit parts of the turbulence (if activated) are summed up

2. Evaluate the part of the residual of the prognostic variables (see Equation 6) which does not depend on the next time level ($\underline{\mathbf{U}}^{t+\Delta t}$)

3. The implicit (i.e. using a Crank-Nicholson scheme) vertical turbulent tendencies are added to the previously evaluated tendencies

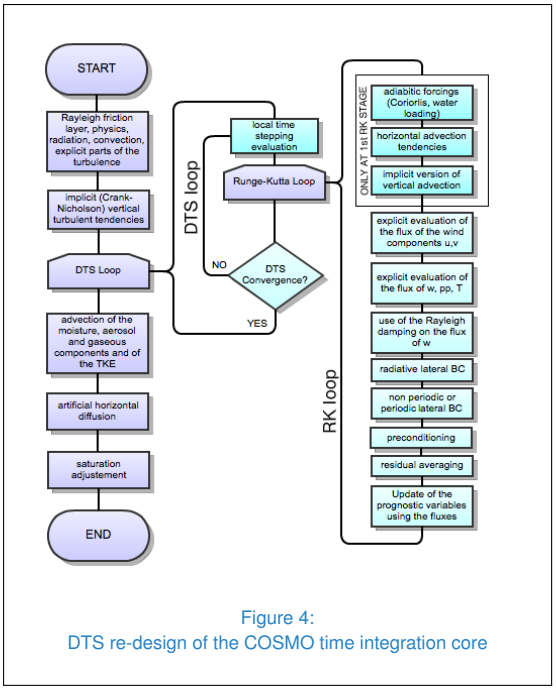

Figure 4:
DTS re-design of the COSMO time integration core

Source: DTS Flow Chart

4. Start of the Dual Time substeps (while a convergence treshold or the maximum number of DTS steps is reached):
   (a) Evaluate the local time step
   (b) Start of the Runge-Kutta substeps:
      i. (*might be only in the first RK substep*) calculate the horizontal advection tendencies
      ii. (*might be only in the first RK substep*) if the implicit vertical advection is used then, using the previously calculated tendencies from horizontal advection and the above mentioned tendencies due to physics and adiabatic processes of the dynamic variables, the vertical advection is solved by a vertically implicit scheme.
      iii. evaluate the complete fluxes *(adding the previously evaluated slow modes to the fast modes)* of u and v using an explicit scheme
      iv. evaluate the complete fluxes *(adding the previously evaluated slow modes to the fast modes)* of w, pp and T using an explicit scheme instead of the implicit Crank-Nicholson scheme used in the original fast solver
      v. apply the Rayleigh damping to the flux of the w-velocity and the lateral BCs to all the prognostic variables
      vi. complete the residuals of the prognostic variables by adding the previously evaluated complete fluxes to the incomplete residuals containing the fixed part of the time derivative (see step 3)
      vii. apply the preconditioning
      viii. apply the residual averaging
      ix. update the prognostic variables
   (c) end of the Runge-Kutta substeps
5. end of the Dual Time substeps with the convergence to a fictitious steady state.

   Advection of the moisture fields (q_x), aerosol, gaseous components and the Turbulent Kinetic Energy (TKE) if active
6. In an operator splitting manner perform the horizontal diffusion of q_x and the implicit vertical diffusion of q_v (specific water vapor content), q_c (specific cloud water content), q_i (specific cloud ice content) and TKE
7. apply artificial horizontal diffusion to the prognostic variables
8. call the saturation adjustment

TEST CASE

As the aim of this work is to assess the feasibility of a DTS time integration core in COSMO, the solutions obtained with a preliminary version of DTS and the original code for a simple test case of mountain flow are compared. The mountain ridge has a gaussian shape

$$h(x) = He^{-\frac{x^2}{a^2}log2} \qquad (15)$$

where H is the mountain maximum height and a (the mountain width) is the horizontal distance from the mountain top where the height is $H/2$.

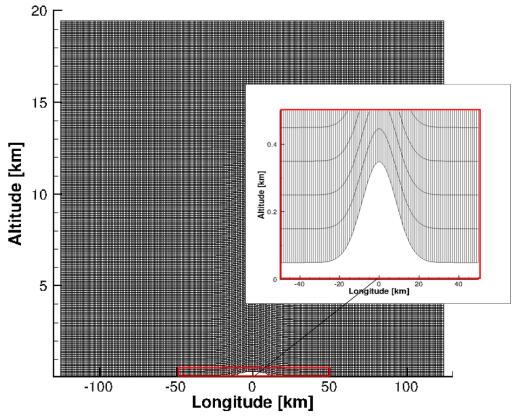

Figure 5:
Grid adopted for the mountain flow and detail of the SLEVE vertical coordinates near the mountain ridge

Source: Computational domain

The computational domain is 250 km large in x-(horizontal) direction and 19.5 km in z-(vertical) direction. The mountain ridge is located in the middle of the longitudinal field, as shown in Figure 5. Keeping in mind the setting described in Table 1, first simulations have been performed with the original COSMO code.

The w-velocity component after to 50 hours of simulation is shown in Figure 6. It is possible to locate the position of the mountain in the middle of the domain from the waves.

Table 1
Simulation parameters: mountain flow

	Value
x-resolution, Δ x	1 [km]
y-resolution, Δ y	1 [km]
z-resolution, Δ z	0.1 [km]
N^o of cells in x-direction	250
N^o of cells in y-direction	7 (2d case)
N^o of cells in z-direction	195
time step	10 [s]
simulation time	50 [h]
advection order	5
divergence damping	0
rayleigh damping	Active for $z > 11000m$
time integration scheme	RK3
free-stream velocity	10 [m/s]
mountain height, H	300 [m]
mountain width, a	10 [km]

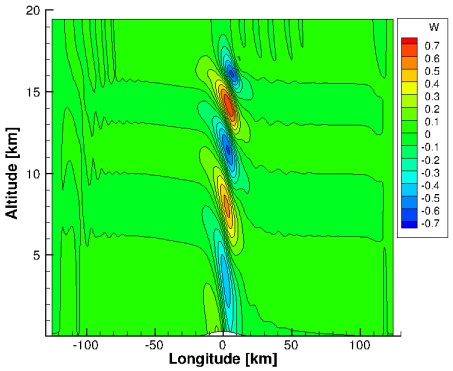

Figure 6:
Contour of w-velocity

Source: Original COSMO

In the case of the DTS simulations the additional settings required are summarized in Table 2 . The w-velocity component obtained with the DTS version of the COSMO software up to 50 hours of simulation is shown in Figure 7 .

The threshold chosen for the convergence of the dual time iterations is the decay of two (logaritmic) levels in the maximum residual of the w-component of the velocity, as reported in Figure 8. It is possible to notice the typical behaviour

Table 2
DTS additional settings

	Value
CFL of the DTS time integration	10
N^o of Runge-Kutta stages	5
Maximum number of DTS iterations	200
Residual Decay Threshold (logaritimic)	2
Parameter for residual smoothing, RESAV	2
Preconditioning	Active
Local time stepping	Inactive
Multigrid	Inactive

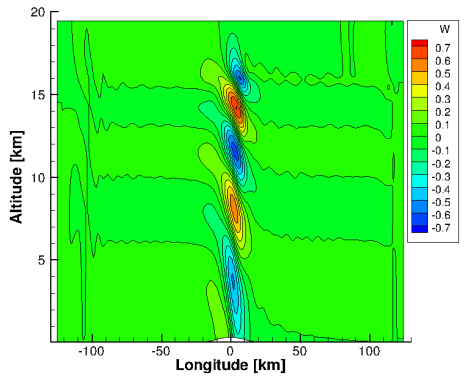

Figure 7:
Contour of w-velocity

Source: DTS COSMO

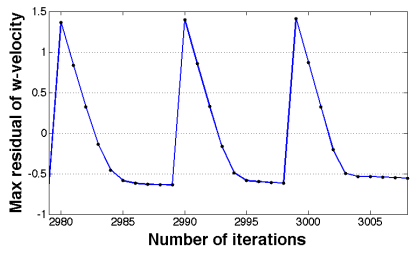

Figure 8:
Convergence history in the dual time

Source: DTS COSMO

of the DTS integration, where each time level is considered as the steady-state solution of a fictitious integration using a pseudo-time. The average number of DTS iterations needed to achieve the two-levels decay is 17 in the reported case, but further achievements are expected using a local time step and the multigrid technique.

CONCLUSIONS

In this paper a DTS time integration core is proposed for COSMO. The paper describes the re-design of the existing routines needed to successfully implement this technique. In the proposed configuration, the convergence acceleration techniques (i.e. local time stepping, multigrid, preconditioning, residual smoothing) for steady solvers can be implemented: a preliminary version of the new solver using preconditioning and residual smoothing was tested. In future works the numerical perfomances of the DTS will be assessed in comparison with the operational RK2 scheme. The proposed test case shows that the DTS integration is able to give results close to the solution obtained with the existing COSMO, leading the author to further investigate the advantages and disadvantages of the proposed methodology with real test cases in Metereology and Climatology.

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