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Modelling Negotiated Decision Making: a Multilateral, Multiple Issues, Non-Cooperative Bargaining Model with Uncertainty

Carlo Carraro

University of Venice, Fondazione Eni Enrico Mattei and EuroMediterranean Center on Climate Change, CMCC

Alessandra Sgobbi

Fondazione Eni Enrico Mattei and EuroMediterranean Center on Climate Change, CMCC

Centro Euro-Mediterraneo
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Summary
The relevance of bargaining to everyday life can easily be ascertained, yet the study of any bargaining process is extremely hard, involving a multiplicity of questions and complex issues. The objective of this paper is to provide new insights on some dimensions of the bargaining process – asymmetries and uncertainties in particular – by using a non-cooperative game theory approach. We develop a computational model which simulates the process of negotiation among more than two players, who bargain over the sharing of more than one pie. Through numerically simulating several multiple issues negotiation games among multiple players, we identify the main features of players' optimal strategies and equilibrium agreements. As in most economic situations, uncertainty crucially affects also bargaining processes. Therefore, in our analysis, we introduce uncertainty over the size of the pies to be shared and assess the impacts on players' strategic behaviour. Our results confirm that uncertainty crucially affects players' behaviour and modify the likelihood of a self-enforcing agreement to emerge. The model proposed here can have several applications, in particular in the field of natural resource management, where conflicts over how to share a resource of a finite size are increasing.

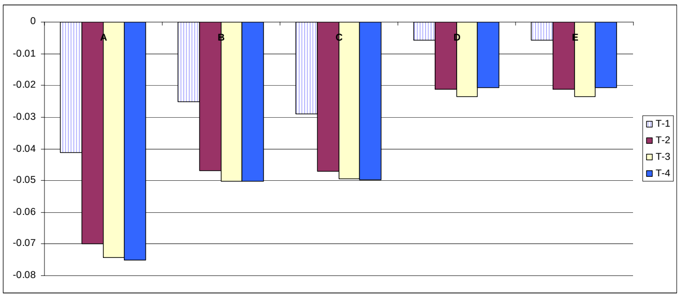
Keywords: Bargaining; Non-Cooperative Game Theory; Simulation Models; Uncertainty.

JEL Classification: C61, C71, C78

Address for correspondence:
Alessandra Sgobbi,
Fondazione Eni Enrico Mattei
Corso Magenta, 63
I-20123 Milan, Italy
E-mail: alessandra.sgobbi@feem.it

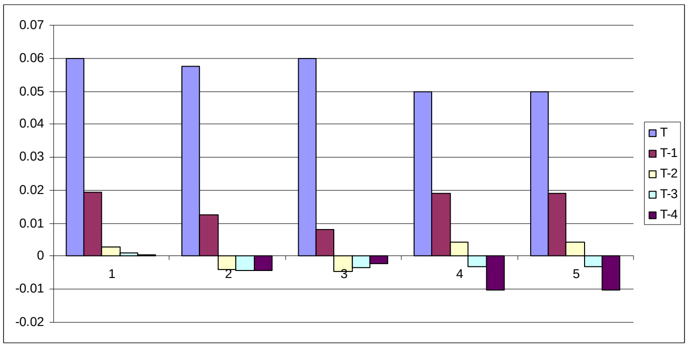
Figure 6 shows the changes in utility level for each player when they are selected to be the proposers in the respective negotiation rounds – that is, it is the difference in utilities they enjoy when proposing an agreement in the restricted and baseline cases. It is clear that all players experience a decrease in utility as compared to the baseline case. The decrease is however not uniformly affecting all players: those who have stronger preferences for the restricted policy issue (that is, those with a higher ideal point for X_2, players A, B, and C) suffer more from this restriction than the other players. The decline in utility is mitigated by the different weights that individual players assign to X_2 relative to X_1, as indicated by the different values of $\eta_{i,k}$ in Table 1: thus, players A and B, who have a stronger preference towards X_2 as compared to X_1, suffer a loss higher than player C, who has a higher preferred point for X_2, but assigns a low weights to this variable relative to the previous two players. This result of the simulation exercise is in line with both the theoretical findings of non-cooperative bargaining theory and the applications of non-cooperative bargaining models to water negotiations (see Carraro et al., 2005, and Carraro et al., 2007). Furthermore, more iterations are needed before a limit point equilibrium solution is found, indicating the increased difficulties in finding a compromise allocation.

Figure 6: Changes in players' utilities from restricting the issue space



Interestingly, the players with a higher ideal point for the restricted issue will "bargain harder" in the last rounds of the negotiation game, and require a higher share of the total resource available for themselves. This effect is shown in Figure 7: should the final round of the negotiation game be reached, the first three players, when selected to be proposers, will ask for themselves a higher share of X_2. The effect decreases for player C as the game proceeds backward, because of the larger opportunity that this player has to compensate for losses in X_2 with higher X_1. These results are robust to further restriction in the issue space.

Figure 7: Change in players' proposed shares – x_2



Consider now the case in which one of the player's preferences towards the two negotiated variables changes in such a way that he now strongly prefers satisfying his ideal quantity of one negotiated variable relative to the other. Intuitively, one would expect the equilibrium shares of this player to change so that his equilibrium quantity of the strongly preferred variable is higher than in the baseline simulation exercise.

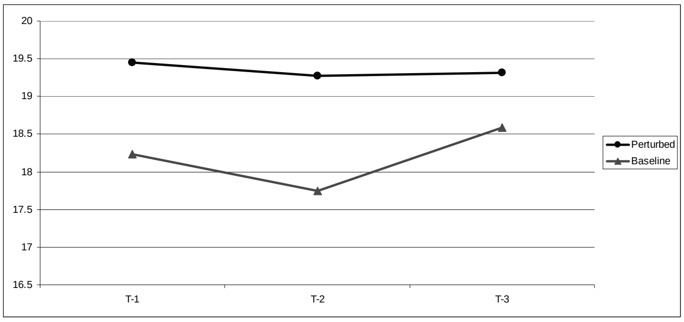
In the simulation exercise, we change the relative weights that player D assigns to X_2 relative to X_1, as shown in Table 6.

Table 6: Changing the relative importance of the negotiated variables.

Player D	$\eta_{D,1}$	$\eta_{D,2}$
Initial	0.5	0.5
Perturbed	0.2	0.8

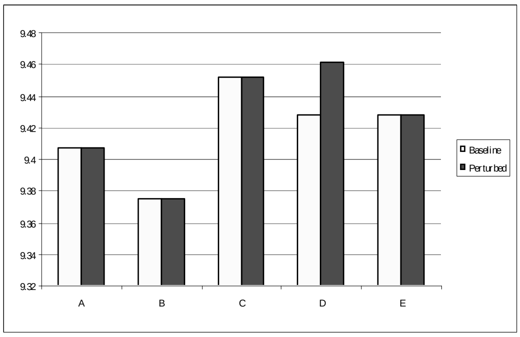
As expected, player D will require a higher share of X_2 relative to the baseline case (see Figure 8).

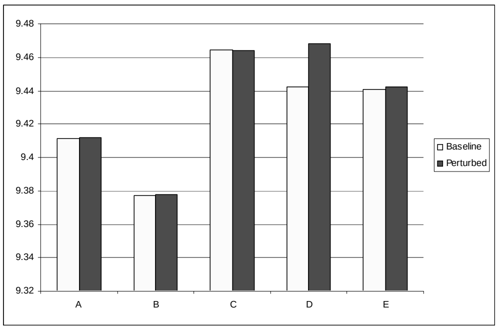
Figure 8: Variation in player D proposals for x_2



What is of interest is that his stronger position affords player D a stronger bargaining position, and his participation constraints in round $T-1$ tightens, while the participation constraints of the remaining players are substantially unchanged (Figure 9, $T-1$). This effect is preserved through the backward induction process, and in equilibrium player D will enjoy a higher utility level (Figure 9, Equilibrium). In this model, there seem to be two sources of bargaining power: first of all, players' access – which, however, is neither linearly nor monotonically related to players' equilibrium payoffs – and players ideal points – both in terms of their magnitude and relative importance.

Figure 9: Changes in players' participation constraints in round T-1





T-1 Equilibrium

Finally, it is interesting to note that, in the long run, a relatively large change in players' weighting of the negotiated variables leads to utility levels that are significantly higher for that player, but leave the expected utilities of other players substantially unchanged.

The role of uncertainty

One of the key aspects of negotiation processes is uncertainty over the size of the negotiated variables (the size of the pie). Let us analyse how this type of uncertainty affects the agreement and players' utilities. In the numerical analysis, we look at the impact of introducing a random component in the constraint function for $\overline{X}_2$. The new constraint for this variable will thus take the following form:

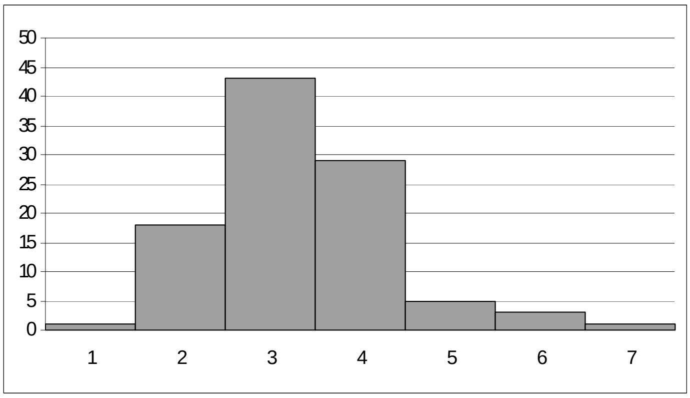
[7] $$\sum_N x_{i,2} \leq \tilde{X}_2$$

where $\tilde{X}_2$ is an uncertain component of the size of the pie to be divided. In the case of negotiations on water availability, for instance, the total quantity available depends in part on precipitation levels, which, however, cannot be predicted with certainty.

To demonstrate the relationship between the introduction of uncertainty in the realisation of one of the negotiated variables and the frequency of different solution, we report the results of a Monte Carlo experiment, in which we solve the model for 100 randomly drawn values of $\tilde{X}_2$, assuming an exogenously specified underlying probability distribution for the unknown term. Using random inputs, the deterministic model is essentially turned into a random model.

The choice of the underlying distribution to simulate random sampling may impact the results of the simulations. For the numerical example, we will assume that the random component of $\overline{X}_2$, $\tilde{X}_2$, is drawn from a gamma probability distribution[10], with shape parameter 13 and scale parameter 8.5. The corresponding mean and standard deviations are, respectively, 104.4 and 29.3. Figure 10 shows the frequency distribution of the realised values of $\tilde{X}_2$, together with some basics statistics.

Figure 10: Distribution of X_2



Mean	104.424
Standard error	2.928
Median	97.601
Standar deviation	29.280
Variance	857.328
Curtosi	2.350
Asymmetry	1.210
Interval	165.274
Minimum	48.587
Maximum	213.861
Count	100.000

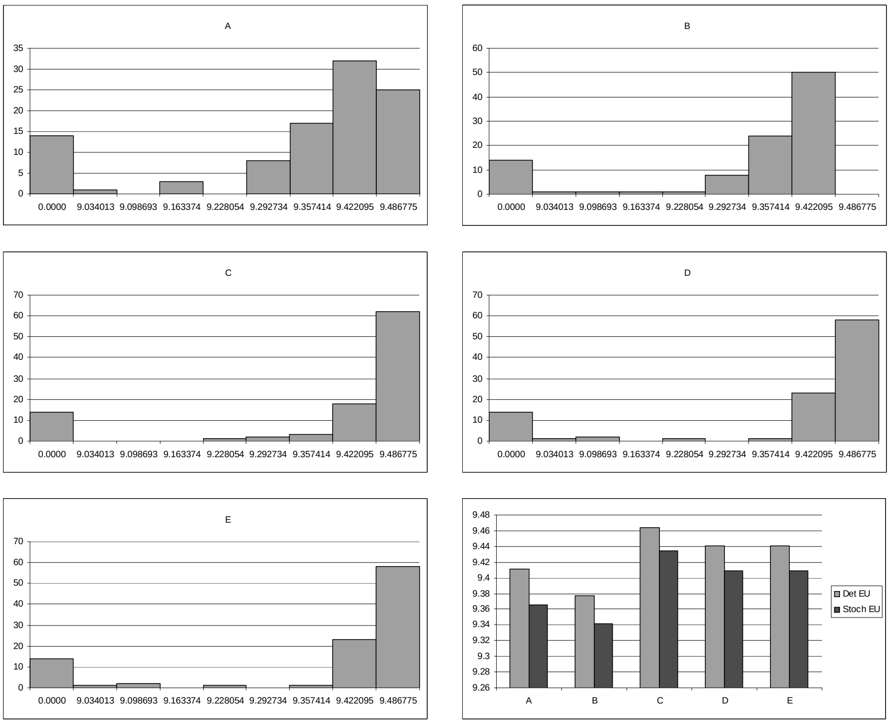
[10] There are various types of probability distribution one could use: the normal distribution is applicable to variables whose values are determined by an infinite number of independent random events; very rare events are best represented using the Poisson distribution. The normal distribution is symmetrical around the mean and, in general, it is used when (i) there is a strong tendency for the variable to take a central value; (ii) positive and negative deviations from the central value are equally likely; and (iii) the frequency of deviations falls off rapidly as the deviations become larger. The gamma distribution is, on the other hand, widely used in engineering to model continuous variables that are always positive and have a skewed distribution.

For each of the 100 sampled bargaining problems, we examine the emerging strategies of players, as well as the equilibrium solutions and utilities, for 100 bargaining rounds.

From the results, it would appear that *it takes longer to find an agreement when uncertainty over the resource is included*. Moreover, in 14% of the cases, a feasible agreement cannot be achieved – that is, the equilibrium offers of players are not compatible with the resource constraint.

The first 5 histograms of Figure 11 show the frequency distribution of players' utilities when the size of negotiated variable is uncertain – so, for instance, player D and F experience more frequently low utility levels when all the players entertain the possibility of variations in the size of one pie. The variability of players' equilibrium utilities differs among the players, while for all of them the equilibrium (limit) utility is lower in the stochastic case than in the deterministic case, as shown by the lower right quadrant of Figure 11.

Figure 11: Frequency distribution of players' equilibrium utilities



Of course, the deterministic case assumes a known realisation of the pie. But what happens to players' utilities if this belief is mistaken? Would they be better off by considering this possibility in their bargaining strategy?

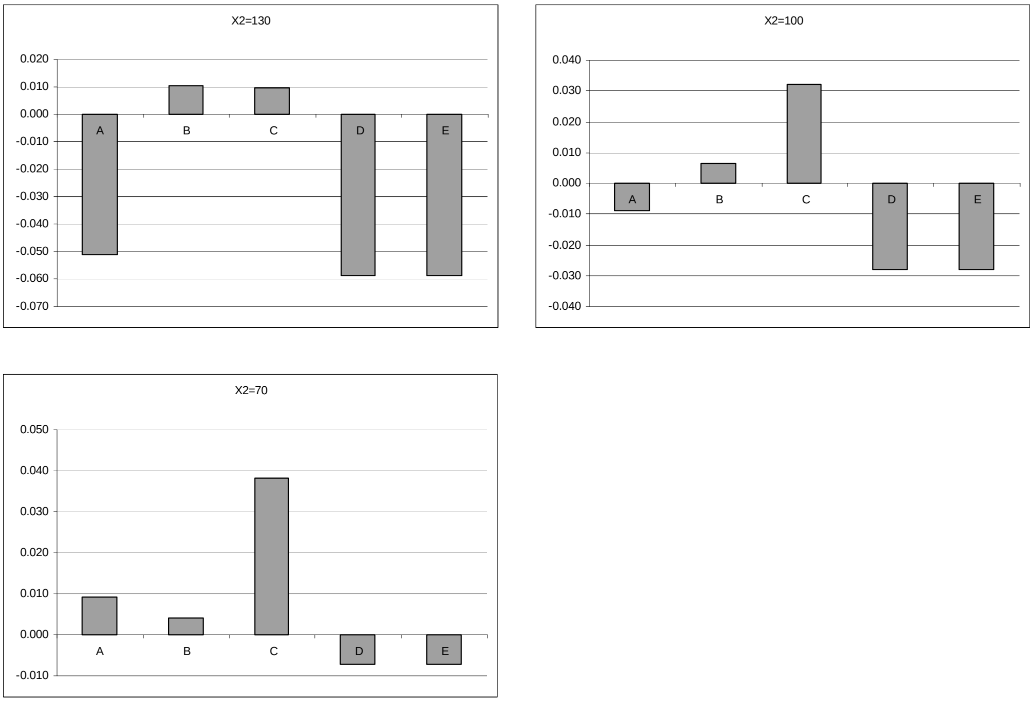
In order to assess the ex post efficiency of the different negotiation frameworks, let us compare the utilities that player would derive, in equilibrium, from the negotiated agreement under the deterministic and stochastic case. Table 7 below reports the equilibrium shares agreed upon in the deterministic and stochastic models.

Table 7: Equilibrium shares

X1	A	B	C	D	E
Deterministic	0.2002	0.2002	0.2013	0.2002	0.2002
Random	0.2026	0.1986	0.2027	0.2018	0.2018
X2					
Deterministic	0.2004	0.2001	0.2014	0.1985	0.1985
Random	0.1997	0.1997	0.2015	0.1997	0.1997

As shown in the first left hand side quadrant of Figure 12, players do, in general, better in the deterministic type of model as compared to the stochastic type. However, as the resource available begins to shrink, players' utility increases, in general, when they take into account the uncertainty surrounding the realisation of the surplus. The explanation is intuitive: *as players begin to account for uncertainty in their strategy, they will try to negotiate harder*, expecting a higher share of the surplus, in order to increase their chance of coming closer to their ideal[11].

Figure 12: Changes in players' equilibrium utility – deterministic vs. stochastic



[11] Note that, because of the construction of our preference function, an excess allocation of water is a punishment for players. This may not be realistic in some circumstances, as discussed more in details in the concluding section.

Conclusions

The model proposed in this paper attempts to simulate the process of negotiation among multiple players, who have to decide on how to share a surplus of fixed size. In this context, negotiation rules are simulated through an offer and counteroffer procedure. Players have payoff functions that depend on the share of the surplus that they can secure for themselves – with different negotiated variables having different importance for each player, thus generating space for tradeoffs among them. Furthermore, players have varying access probabilities, which signal the relative strength at the bargaining table and thus influence the equilibrium agreement.

Through a series of simulations in which five players negotiate over the respective shares of two cakes, we have examined the emerging equilibrium agreements and their characteristics. What can be inferred about these problems by applying numerical simulation?

First of all, the results conform to expectations when there is no uncertainty over the negotiated variables. As in the Rausser-Simon model and its applications, increasing the access probability of a player will yield outcomes that are more favourable for the "more powerful" player, but also to players with similar preferred positions. Convergence of the solution is attained in few iterations of the model – which can in part address some of the critiques moved to backward induction, as there is scepticism of long and involved inductive chains.

This result does no longer hold when we restrict significantly the range of admissible values for the negotiated variables. In fact, restricting the size that the negotiated variables can take reduces the opportunities for trade, yielding potentially lower utilities to all players. In some cases, excessively reducing the boundaries of the negotiated variable may shrink the bargaining space so much that no zone of agreement remains. Should this result emerge when exploring a real problem using this framework, it would be advisable to attempt changing the decision rule – from unanimity to qualified majority, for instance.

Importantly, the effect of bargaining power on the equilibrium agreement is non linear, but rather evolve in complex way through the process of backward induction. The effect of shifting access depends crucially on other constitutional factors with which it interacts – such as decision rules, players' preference parameters, the relative distance of their ideal points, so on and so forth. Thus, there are synergies among players or issues that affect the ultimate impact of bargaining, contrary to the assumption of the standard Nash games.

Finally, uncertainty over one of the negotiated variables crucially affects the equilibrium outcome and the players' strategies. Our main results are:

(i) when uncertainty is introduced, the negotiation takes, on average, longer (14 rounds as opposed to 7 rounds in the deterministic case);

(ii) in some cases, players' strategies do not even converge to a feasible solution – that is, players' offers crystallise on values that are not compatible with the resource constraint for neither variables;

(iii) explicitly accounting for uncertainty in the realisation of the surplus leads, under some circumstances, players to bargain harder: ex post, they are better off only when the realisation of the surplus is low, as compared to the deterministic case.

These results are in line with intuition and with previous results of similar models: they therefore lend support to the hypothesis that non-cooperative bargaining is a useful framework for exploring negotiation processes and players' strategic behaviour. Applying non-cooperative bargaining theory can provide some useful insights, based on formal models, as to which factors influence to a significant extent players' strategies and, as a consequence, the resulting equilibrium agreement policy.

The value added of exploring management problems within a non-cooperative bargaining framework lies in the ability of the approach to help finding politically and socially acceptable compromise. The proposed model can find several applications, in particular, in the field of natural resource management – where conflicts over how to share a resource of a finite size are increasing.

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